On the comparison of possibilistic and probabilistic methods for the determination of the worst-case response of mistuned bladed disks

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ABSTRACT

In this paper, the possibilistic and probabilistic methods are compared with each other by utilizing a lumped parameter vibratory system and a mistuned bladed disk model. In these models stiffness elements are considered to be uncertain, which are assumed to have Gaussian probability distribution. Both methods are utilized for determining uncertain natural frequencies and the worst-case blade tip displacements. The blade tip displacement amplitudes gathered from different number of Monte-Carlo iterations and calculated by fuzzy forced response solution are compared and it is found that among the two, only the possibilistic method converges to the worst-possible blade tip displacement value, and requires significantly less computational time.

1 INTRODUCTION

Mistuning which occurs from destruction of cyclical symmetry in bladed disk assemblies due to residual uncertainties is a well-studied phenomenon in vibrations of rotating machinery. It has been investigated under several sub-topics in engineering discipline; such as identification, determining the worst and the best mistuning pattern, forced response reduction and structural health monitoring. The performance of all these investigations boils down to the true quantification of uncertainties that are present in the engineering system [1-4]. In this paper, performances of the two approaches to handle uncertainties; i.e. probabilistic and possibilistic methods are compared for their reliability and computational effectiveness on determining the worst-case scenarios of bladed disk assemblies.

The favorable areas for the two interchangeable methods have been investigated by several researchers and Maglaras et.al. [5] reviewed the state of the art in literature. In the review paper, the authors stated the common conclusion that probabilistic methods favor in situations where there is enough information regarding uncertainties, and possibilistic methods favor in situations where there is lack of information and in situations where the worst-case scenarios are sought. In this paper, a conclusion parallel to the common conclusion in literature is drawn by utilizing Monte-Carlo methods for probabilistic analysis and fuzzy modal analysis method for possibilistic analysis for the same problem. Possibilistic methods are found to be more reliable and computationally more efficient in determining the uncertainty range of the worst-case natural frequency and the worst-possible forced response of a mistuned bladed disk model at each and every confidence level.

The methodology that is used to compare reliability and computational performances of fuzzy modal analysis and Monte-Carlo analysis on determining the worst-case response is detailed in the next section and it has been exemplified by two case studies. In the case studies, a simple lumped parameter system and a cyclically symmetric lumped parameter bladed disk model is used to interpret the progress of the reliability performances of the methods. The value of the worst-case blade response is calculated for uncertain stiffness parameters in order to compare the computational effectiveness of the methods.

2 METHODOLOGY

In this study, methods that characterize the same uncertain situation in two different ways are compared with each other on the basis of reliability and computational efficiency in determining the worst-case situations in bladed disk assemblies. Probabilistic methods assume uncertainties to be random where the outputs of an uncertain event are precisely measurable but cannot be determined before an experiment is conducted. These methods model uncertain variables as random variables and characterize uncertainties by probability density functions. In this method, expected values of outcomes of uncertain situation are interpreted by processing relative frequency of outputs obtained for the input random set. On the other hand, possibilistic methods are used to account for another type of uncertainty called imprecision which assumes either precision of measurements to be very low or meaning of measurements to be not clear. In this method, uncertain parameters are modeled as fuzzy variables and characterized by possibility distributions [6]. In possibilistic analysis, range of possible values of outcomes are sought and organized for each and every confidence level.

The connection between possibilistic and probabilistic methods are established upon the logical truth stating that '*probability of a measured event cannot be more than possibility of that event*' [5] which is based on possibility/probability consistency theorem proposed by Zadeh [7]. Due to nature of the theorem, interpretation of the connection remains open to variations according to situation at hand. In this paper, uncertain parameters are assumed to deviate around design value of corresponding parameter in the engineering system and also deviations of possible values of uncertain variables are assumed to be bounded. According to this assumption, the design value of the uncertain parameter needs to be assigned to the mean value of the random set that have Gaussian distribution for probabilistic analysis and membership function distribution value of 1 for possibilistic analysis. The assumption further necessitates both distribution functions to share the same uncertainty bounds, as shown in Fig. 1 below.

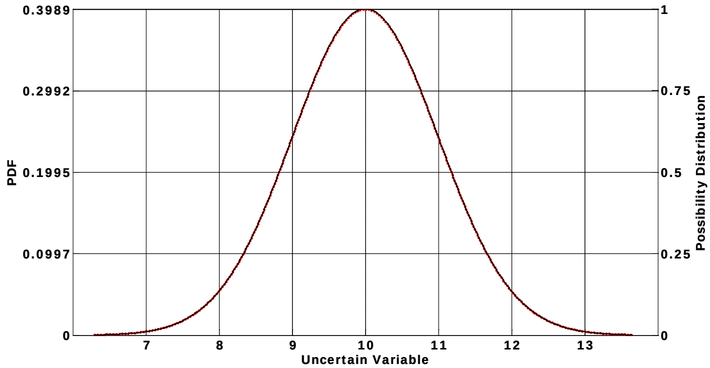


Figure 1: Consistency of the two uncertainty models

The assumption establishes the connection by stating the most expected values and the bounds of uncertainty ranges, only. In order to determine states in remaining levels of confidence, a distribution character of uncertainties needs to be assessed. In this paper, compatible with the mistuned bladed disk systems, stiffness parameters are assumed to have Gaussian distribution characters that is identified by standard deviations and mean values. According to the assumption, design values of stiffness parameters are assigned to the mean value of the corresponding Gaussian distributions and the extent of deviations of uncertain parameters around the design values are represented by assigned standard deviation values.

In probabilistic analysis Gaussian random sets containing values of uncertain stiffness variables are used in Monte-Carlo analysis in order to come-up with random sets of mistuned natural frequencies and values of blade forced responses. Monte-Carlo analysis picks one value from each random set of stiffness variables in each iteration and calculates mistuned outputs by modal analysis. Pursuing iterations for a number of times, creates a random set for each output. The probability density functions of calculated (random) mistuned natural frequencies are used for comparison with the corresponding possibility distributions on the basis of calculation of the uncertainty ranges corresponding to the worst-case natural frequency. Moreover, the worst-case forced response value is determined from the random data set of mistuned forced responses.

Correspondingly in possibilistic analysis, in order to come up with possibility distribution functions of mistuned natural frequencies and the worst-possible blade responses, intervals of confidence levels of possibility distributions (alpha-cuts) are used in extension principle solution of the fuzzy modal analysis. Alpha-cuts are slices of the possibility distributions which assesses expected values that a fuzzy variable can take within the corresponding level of confidence. For instance, for a fuzzy stiffness parameter $\bar{k}$, alpha-cut of 0.4 confidence level is represented by an interval $\bar{k}[0.4]=[k_{lower},k_{upper}]$ meaning that at 0.4 confidence level, stiffness parameter $\bar{k}$ is expected to take values between k_{lower} and k_{upper} . Here, alpha-cuts of possibility distributions of uncertain stiffness parameters are determined from the corresponding probability density functions compatible with the interpretation of consistency principle in this study, which is expressed mathematically as follows

$$\mu(x)=\frac{p(x)}{\max[p(x)]}, \qquad (1)$$

where $\mu(x)$ stands for membership function distribution of the possibility distribution and $p(x)$ stands for the probability density function. Eq. (1) scales probability density functions with its maximum value resulting in a membership value of 1, where both distributions share the same uncertainty bounds. Furthermore, utilizing Eq. (1), alpha-cuts of possibility distributions of fuzzy stiffness variables, used in fuzzy modal analysis, are calculated, an example of which is shown in Fig. 2. It should be noted that, possibilistic method does not require a prior knowledge of distribution of the uncertainty; however, in order to perform a valid comparison the same distribution as in the probabilistic method is used in this study.

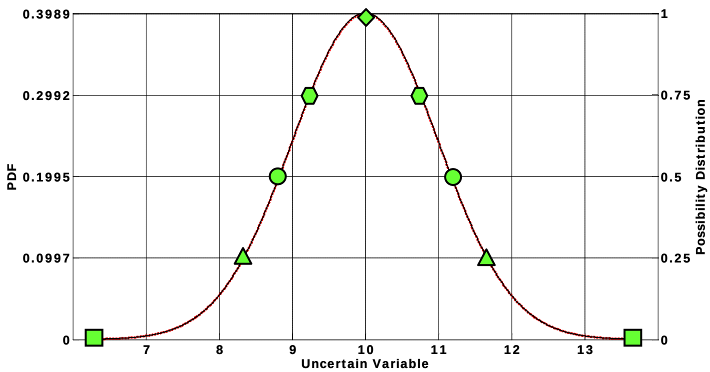


Figure 2: Alpha-cuts of possibility distributions

Alpha-cuts of fuzzy stiffness parameters are processed in extension principle solution of fuzzy modal analysis in order to determine the alpha-cuts of the fuzzy mistuned natural frequencies and the worst-possible blade forced responses within the uncertainty range of interest. Extension principle solution of fuzzy equations is a mapping between inputs and outputs of a fuzzy function that arranges possibility value of the output with respect to the possibility values of the input. It looks for each and every pair of inputs that constitute an output –regardless of their confidence levels- and sets the possibility value of the output accordingly. However, since the physical systems are continuous, in the solution of the worst-case problems, a special case of the extension principle method for the continuous functions is used. This method searches for all pairs of inputs that constitute an output in the same confidence level and assigns the possibility value of the output to possibility value of the inputs [6, 8].

The formulation of extension principle solution for fuzzy natural frequency and fuzzy forced vibration responses are given as follows.

$$
\begin{aligned}
&\overline{\omega}[\alpha]=[\omega_1(\alpha),\omega_2(\alpha)]\text{ , n = 1...DOF of the lumped parameter system}\\
&\omega_1(\alpha)=\min\{\Psi(k_{11},\dots,k_{nn})\,|\,k_{ij}\in\overline{k}_{ij}[\alpha]\}\\
&\omega_2(\alpha)=\max\{\Psi(k_{11},\dots,k_{nn})\,|\,k_{ij}\in\overline{k}_{ij}[\alpha]\}\\
&\overline{q}[\alpha]=[q_1(\alpha),q_2(\alpha)]\\
&q_1(\alpha)=\min\{\Phi(k_{11},\dots,k_{nn})\,|\,k_{ij}\in\overline{k}_{ij}[\alpha]\}\\
&q_2(\alpha)=\max\{\Phi(k_{11},\dots,k_{nn})\,|\,k_{ij}\in\overline{k}_{ij}[\alpha]\}
\end{aligned}
\qquad (2)
$$

In the above formulation, the algorithm Ψ is utilized to calculate alpha-cuts of fuzzy natural frequency, $\bar{\omega}[\alpha]$, which accepts alpha-cuts of fuzzy stiffness variables $\bar{k}_{ij}[\alpha]$ and returns the upper and the lower bounds of fuzzy natural frequencies $[\omega_1(\alpha),\omega_2(\alpha)]$ at the same confidence level. Likewise, the algorithm Φ calculates alpha-cuts of fuzzy forced response which accepts again alpha-cuts of fuzzy stiffness variables $\bar{k}_{ij}[\alpha]$ and returns the upper and the lower bounds of fuzzy forced response $[q_1(\alpha),q_2(\alpha)]$. These algorithms utilize modal analysis methods and use genetic algorithm in order to process min/max operations.

The ensemble of alpha-cuts of fuzzy natural frequencies constitutes the possibility distribution of the fuzzy natural frequencies and the resulting possibility distribution character is compared with character of the probability distribution obtained by probabilistic analysis for reliability considerations. Furthermore, the upper bound of the fuzzy forced response, i.e. parameter $q_2(\alpha)$, gives the worst-possible blade forced response in each and every confidence level, and the upper bound obtained for the least confidence level is compared with the corresponding maximum blade forced response value calculated from Monte-Carlo simulations.

The use of alpha-cuts in possibilistic methods brings computational advantage, since the calculations can be done iteratively by using the results of the previous level of confidence in the next level of confidence. This opportunity results in an increase in computational accuracy and a decrease in computational time. Together with the reliability based comparison, the advantages of utilization of alpha-cuts are exemplified on two lumped parameter systems in this study. First, a simple lumped parameter system with different number of uncertain variables is used to show the performance of the methods with increasing number of uncertain parameters. Afterwards, a lumped parameter model of a mistuned bladed disk system is considered and the worst-case mistuning considerations are exemplified utilizing all stiffness parameters as uncertain.

3 NUMERICAL EXAMPLES

3.1 CASE I – Simplest Case

The case is designed to exemplify the similarities and differences between both methods by considering different number of uncertain variables. The case utilizes a 10 degree-of-freedom (dof) lumped parameter model as the vibrating system which is excited by sinusoidal excitations. The layout of the 10 dof system is shown in Fig. 3 and the design values of mass and stiffness parameters are given in Table 1 and a structural damping ratio of 0.01 is used in the analysis.

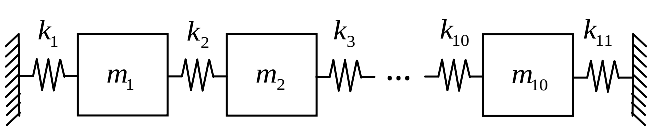


Figure 3: 10 dof lumped parameter model

Table 1: Design Values of 10 dof Lumped Parameter System

Parameter	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	
Value [kg]	2,88	3,45	3,74	7,21	3,40	7,92	3,39	4,59	7,89	3,93	
Parameter	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}
Value [N/m]	43687	32816	46975	45701	30577	41973	46461	43431	35511	44366	31767

The mass and stiffness values given Table 1 are determined random normal distribution around a mean of 2 kg and 30000 N/m, respectively. The system is excited by two sinusoidal excitations of 100N and 200N amplitudes with an excitation frequency of 24.52 Hz (first natural frequency value) at the first and the second degrees-of-freedoms respectively.

In the first part of this case study, values of the stiffness parameters k_1 and k_2 are assumed to be uncertain and have Gaussian distribution where parameters deviate around their design values with 2% standard deviation which corresponds to at most ±8.5% deviation from the design value. In the second part of this case study, all stiffness parameters are assumed to be uncertain having a normal distribution with a standard deviation of 2%.

3.2 CASE II – Worst-Case Scenarios in Bladed Disk Assemblies

The second case study is designed to compare performances of the methods on a cyclically symmetric structure, i.e. a lumped parameter bladed disk model. Both methods are applied to determine the worst-case scenarios occurring due to mistuning phenomenon. In a mistuned bladed disk structure, the mistuning is specified by the deviation of mistuned natural frequencies from the tuned natural frequencies; therefore, on the design stage it is important to determine the worst-possible ranges of frequency deviation for reliability considerations at each and every confidence level. Furthermore, it is known that mistuning results in localization of vibration energy on a particular blade; hence, the worst-possible vibration amplitude amplification value is sought in order to determine the extent of mistuning effect on the mistuned structure. In both of these investigations, not only the accuracy, but also computational time requirements are important for engineering purposes.

A cyclically symmetric lumped parameter model with 16 blades shown in Fig. 4 is used as the bladed disk model in the second case study. The disk mass, M, disk stiffness, k_d, blade mass, m and blade stiffness, k_b, are given in Table 2, below. All 32 stiffness values are assumed to deviate around their design value given in Table 2 with a standard deviation of 0.15% which corresponds to at most ±0.8% deviation from the design values. The same bladed disk system with ±5% at most has already been investigated by the authors in a previous study [6] where the bladed disk was assumed to spin in a static pressure field under the effect of the first four engine order excitations. In that study, the third engine order excitation is determined to be the worst case in the frequency range of [57-59] Hz. Since the stiffness values of the bladed-disk model considered here are assumed to deviate at most ±0.8%, which is one sixth of the one in [6], instead of investigating the frequency range of 57 to 59 Hz, response of the system at only a single frequency is considered. This excitation frequency is 57.87 Hz corresponding to the ninth resonant frequency of the tuned model. Third engine order excitation is considered in the analysis, which is determined to be the worst case for the system studied [6].

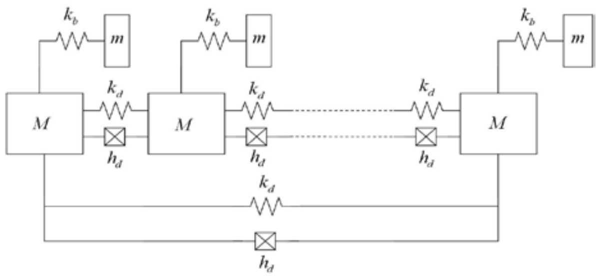


Figure 4: Cyclically Symmetric Lumped Parameter Model of a Bladed Disk System

Table 2: Physical Values of the Cyclically Symmetric Lumped Parameter Model

Parameter	**Value**
Disk Mass	4 kg
Disk Stiffness	60000 N/m
Blade Mass	2 kg
Blade Stiffness	7000 N/m
Structural Damping Coefficient	0.01

4 RESULTS AND DISCUSSIONS

The two case studies are used to exemplify a consistent and a deviated situation on a theoretical model and the behavior of the methods on a model of real-world structure. In all these studies, two important mistuning parameters; the worst-case uncertainty range of mistuned natural frequencies and the worst-case forced response values are sought. The worst-case uncertainty ranges of mistuned natural frequencies are represented by the probability density functions (pdf) in probabilistic analysis and by the possibility distributions in possibilistic analysis. The computed uncertainty ranges in each and every confidence level is obtained by plotting the upper and lower values found on the same abscissa, as shown in Fig. 5 below.

Fig. 5 represents the performance of the methods on a simple theoretical lumped parameter model with two uncertain stiffness values as explained in Case I. It is observed that, in this case, possibilistic and probabilistic methods converges on the same uncertainty ranges. However, this result cannot be generalized, since the performance of the methods deviates as the number of uncertain stiffness parameters increase as shown in Fig. 6 below.

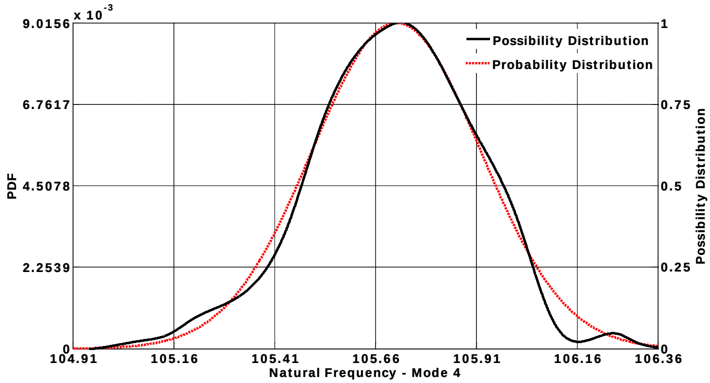


Figure 5: Simple Lumped Parameter System 4th Natural Frequency Distribution- Case I First Part

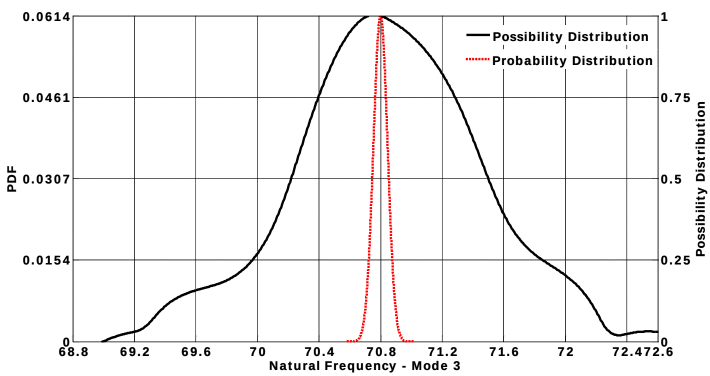
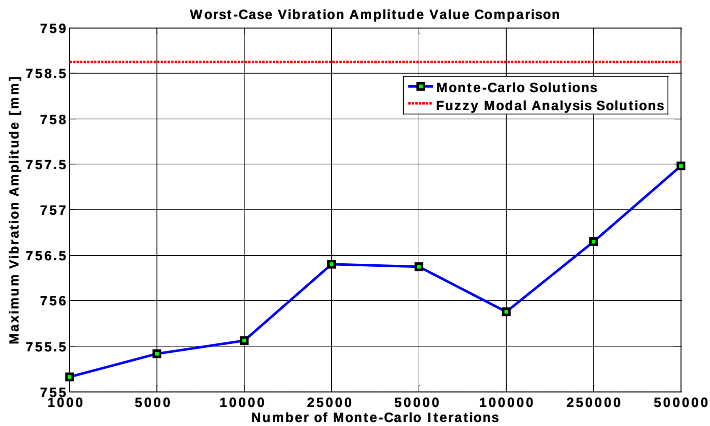


Figure 6: Simple Lumped Parameter System 3rd Natural Frequency Distributions - Case I Second part

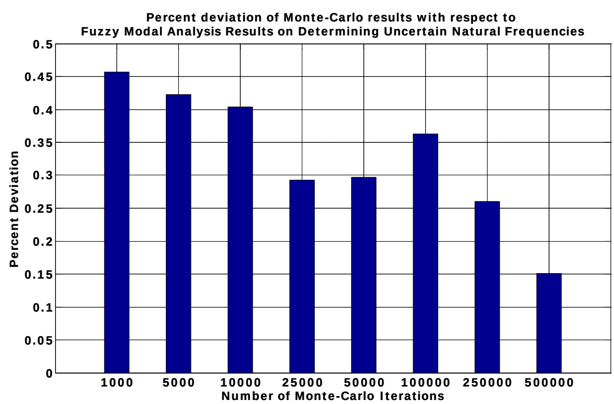
From the results given in Fig.6 it is observed that the possibilistic method gives larger uncertainty ranges at each confidence level compared to the probabilistic method. Therefore, possibilistic methods are found to be more conservative and more reliable in the determination of the worst case. Correspondingly, it can be concluded that probabilistic methods are suitable for the determination of the most probable result whereas possibilistic methods should be used for the identification of the worst-possible case.

A parallel conclusion can be drawn by investigating the performances of the methods in the calculation of the worst-possible forced response values. The results for Case I obtained by fuzzy modal analysis solution and Monte-Carlo analysis with different number of iterations are compared in Fig. 7.

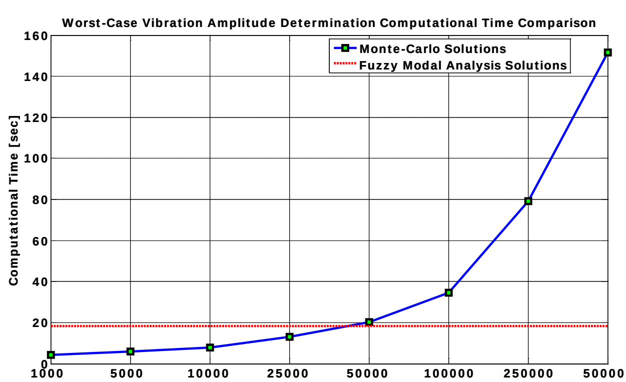
7a)


Worst-Case Vibration Amplitude Value Comparison
Monte-Carlo Solutions
Fuzzy Modal Analysis Solutions
Maximum Vibration Amplitude [mm]
759
758.5
758
757.5
757
756.5
756
755.5
755
1000
5000
10000
25000
50000
100000
250000
500000
Number of Monte-Carlo Iterations

7b)


Percent deviation of Monte-Carlo results with respect to
Fuzzy Modal Analysis Results on Determining Uncertain Natural Frequencies
Percent Deviation
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1000
5000
10000
25000
50000
100000
250000
500000
Number of Monte-Carlo Iterations

7c)


Worst-Case Vibration Amplitude Determination Computational Time Comparison
Monte-Carlo Solutions
Fuzzy Modal Analysis Solutions
Computational Time [sec]
160
140
120
100
80
60
40
20
0
1000
5000
10000
25000
50000
100000
250000
500000
Number of Monte-Carlo Iterations

7d)

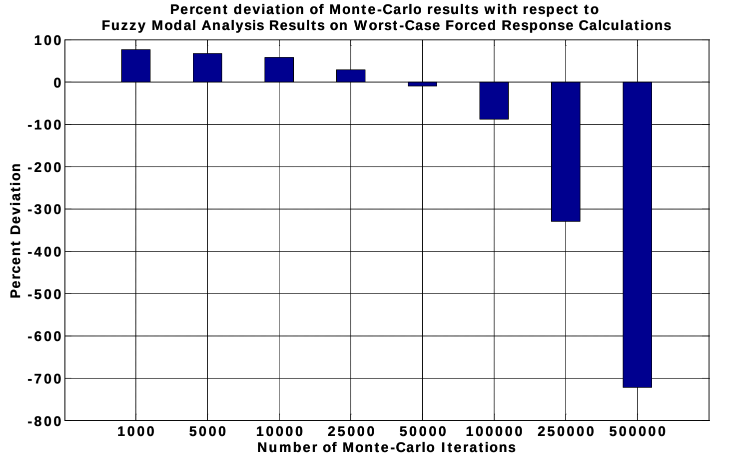
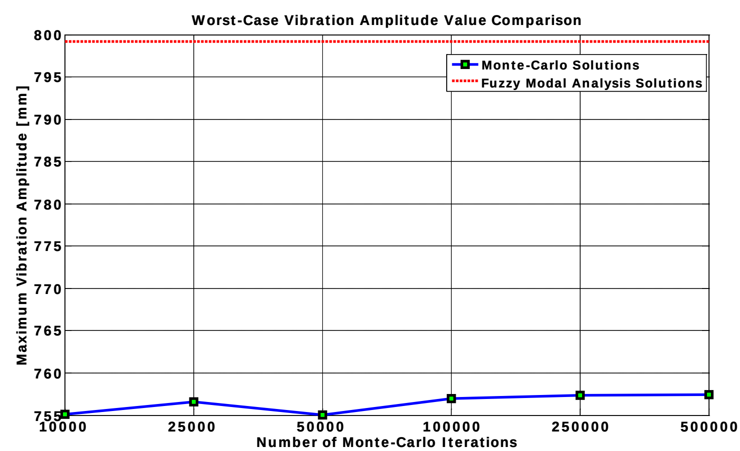


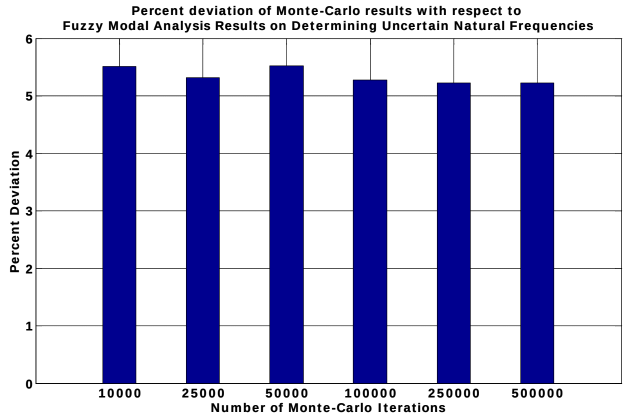
Figure 7: Computational Efficiency Comparison – Case I First Part

From Fig. 7, it is easy to observe that for the system explained in Case I, probabilistic and possibilistic methods can result in the same worst-case forced response value. However, as the number of uncertain parameters increases, the possibilistic methods results in the worst-possible response requiring significantly less computational effort, as shown in Fig. 8. More importantly, increasing the number of iterations in the Monte-Carlo analysis does not improve the results significantly. The error between 100 to 500000 iterations lies approximately at 5%, whereas the computational time required increases more than 700 times compared to the fuzzy modal analysis.

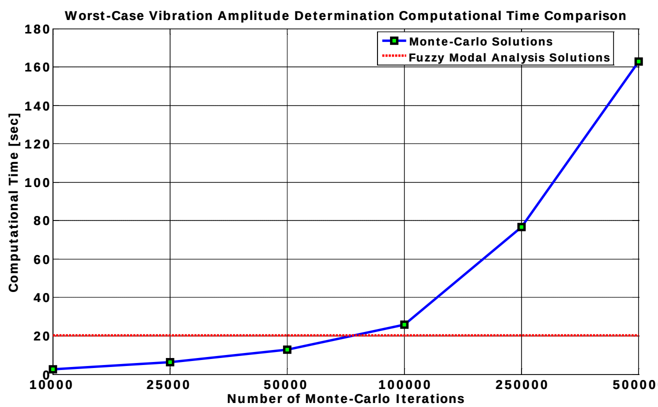
8a)



8b)



8c)



8d)

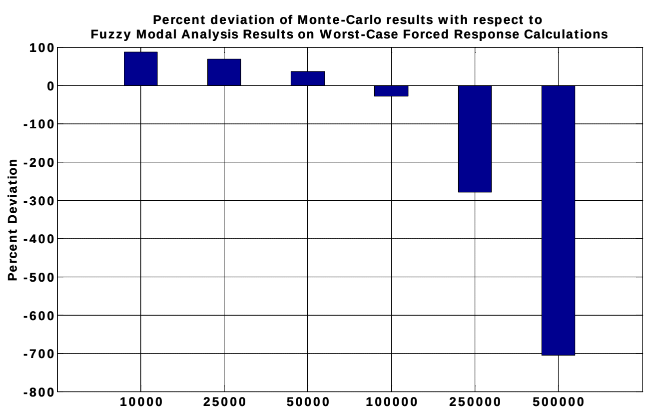


Figure 8: Computational Efficiency Comparison – Case I Second Part

In the final case study, the same analyses are repeated on a bladed disk system with 16-blades. The reliability and computational efficiency comparison results are given in Fig. 9 and Fig 10 below. It is observed that, for the bladed disk system, even though the standard deviation is much smaller than the first case study, the error between the probabilistic and possiblistic methods is higher in the range of 11% to 8%. Moreover, increasing the number of iterations in Monte-Carlo analysis does not result in a significant improvement in the value of the worst response whereas the computational effort required increases more than 6000 times. This is due to the fact that the number of uncertain parameters is nearly 3 times of the first case study.

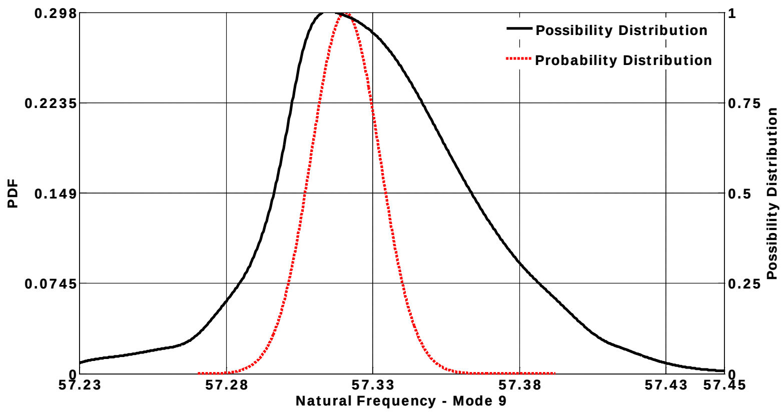
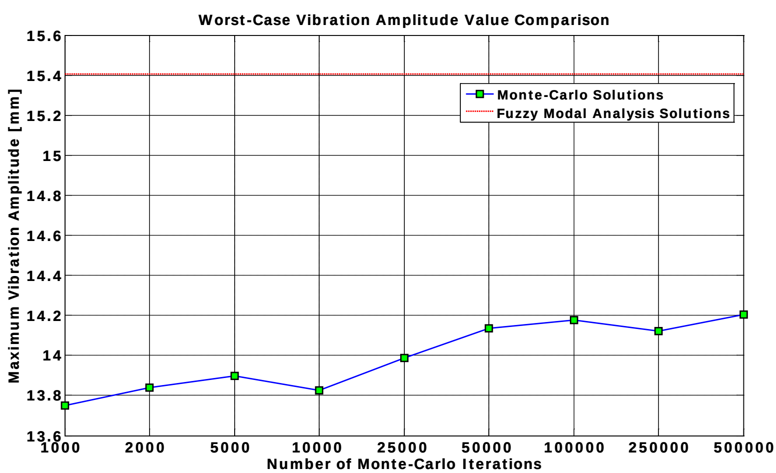
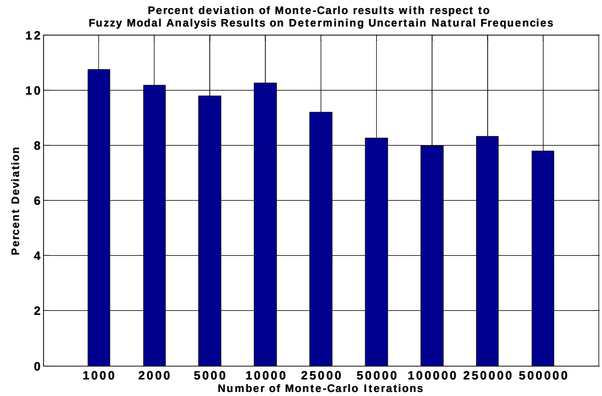


Figure 9: Mistuned Bladed Disk Model 9th Natural Frequency Distribution

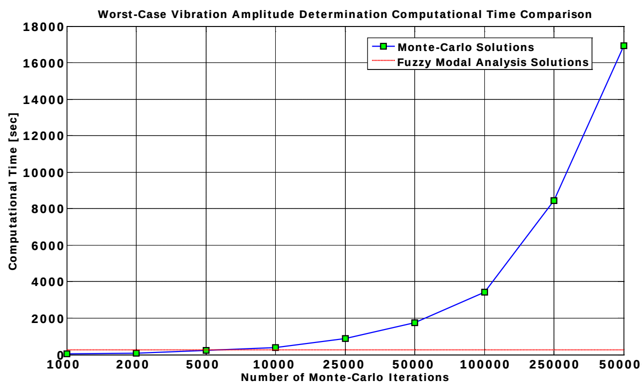
10a)



10b)



10c)



10d)

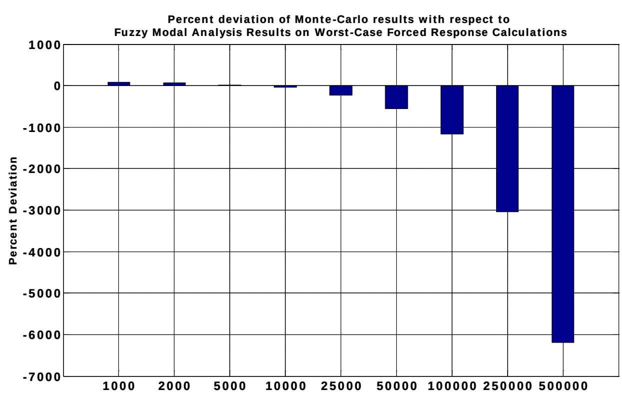


Figure 10: Computational Efficiency Comparison – Cyclically Symmetric Case

5 CONCLUSIONS

In this paper, the possibilistic method (fuzzy modal analysis method) and probabilistic method (Monte-Carlo analysis) are compared in the determination of the worst-case response of a mistuned bladed disk system. Both methods are compared initially using a simple system having only 2 and 11 uncertain parameters. The results show that as the number of parameters increase, in order to obtain the correct worst case response, the number of iterations in the Monte-Carlo analysis should be increased significantly which increases the computational time required drastically. The same comparison is repeated on a lumped parameter bladed disk model with 16 blades considering all 32 stiffness parameters as uncertain. It is observed that increasing the number of iterations in Monte-Carlo analysis from 100- 500000 decreases the error in the worst-case response from ~11% to ~8%. However, utilizing the possibilistic methods, the worst case blade response can be obtained requiring significantly less computational time. Moreover, it is observed that the number of uncertain parameters significantly affects the number of iterations required to obtain an accurate worst case blade response. Therefore, in the analysis of a realistic bladed disk system, where the number of uncertain parameters is as well high, using possibilistic methods decreases the computational effort significantly while resulting in the correct solution.

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